

Investigation of the effect of composite patch dimensions and adhesive layer thickness on the efficiency of one-sided and two-sided repair of cracked plate

Sadegh Daryaei

Graduated in Mechanical Engineering, Kurdistan University, Sanandaj, Iran

sadegh.daryaii@gmail.com

Abstract

In this study, to investigate the effect of composite patch dimensions and adhesive thickness on the performance of cracked plate repair, cracked aluminum plate repaired with composite patches in one-sided and two-sided repair modes have been modeled in 3D in Abaqus software. In this study, by considering different dimensions for composite patches made of boron, graphite, carbon, and glass epoxy, as well as different thicknesses for the adhesive layer, the effect of patch dimensions and adhesive thickness on repair efficiency has been investigated. The results of this study show that for all four composite patches, increasing the patch thickness in one-sided and two-sided repair reduces the stress intensity factor and increasing the patch length increases the stress intensity factor in both one-sided and two-sided repair modes and also, the results show that increasing the patch width in one-sided repair increases the stress intensity factor and in two-sided repair reduces it. Regarding the effect of adhesive thickness, the research results show that increasing the adhesive thickness increases the stress intensity factor and decreases the maximum Von Mises stress in the adhesive layer.

Keywords: stress intensity factor, composite patch, two-sided repair, cracked plate, patch dimensions

1- Introduction

There are cracks in many structures and parts in different industries and the growth of cracks in these structures and parts may lead to failure in parts and ultimately complete destruction of the structure. Therefore, to prevent the growth of cracks in structures, if for some reason it is not possible to replace parts or it is not economically viable, cracks in damaged parts are repaired in various ways such as stop holes, welding the crack, shot peening [1,2], composite and metal patches [3] and filling cracks with glue [4].

Composite patch repair has many advantages over other crack repair methods. Repair with composite patches does not require drilling, and this advantage of composite patches is one of the most important advantages because drilling during repair reduces the strength of the structure. Also, the thickness required for repair with composite patches is about 33 to 50% of the thickness of aluminum patches [5]. Another advantage of repairing with composite patches is that the structure repaired with composite patches does not increase its weight. Structures repaired with composite patches almost double their life and in repair with composite patches, the possibility of parts failure is reduced [6]. Repair with composite patches increases the remaining strength and also increases the fatigue life [7]. Composites have a high strength-to-weight ratio and are resistant to damage and corrosion [8]. Composites are widely used for crack repair in various industries such as aerospace structures, ships, tanks, pipes, etc., and the use of composite patches for crack repair in underwater structures and ships in research [9, 10] has been examined.

In repair with the high-strength composite patch, the stress in the patch itself increases but the stress in the repaired plate decreases [11]. Composite patch thickness is one of the important factors to increase patch efficiency [12]. As the thickness of the composite patch increases, the stress intensity factor decreases, but increasing the adhesive thickness increases the stress intensity factor [3]. Repair with composite patches significantly reduces the stress intensity factor, but the longer the crack length, the faster the stress intensity factor increases [13]. The effect of patch dimensions in different crack angles on the efficiency of one-sided and two-sided repair of cracked plate has been investigated in the study [14], and the results of this study have shown that for zero crack angle, increasing the length of the composite patch in both one-sided and two-sided repair modes increases the stress intensity factor, and increasing the patch width in one-sided repair mode increases the stress intensity factor, but in two-sided repair has reduced it. In research [15], it is recommended that in order for the crack to be under the patch during growth and not to come out of the patch soon, it is better to use a patch with a width much larger than the length of the crack. As the thickness of the adhesive increases, the stress intensity factor increases, but the stress in the adhesive decreases, and therefore the durability of the patch increases [16]. In the design of composite patches, it must be ensured that the stress in the adhesive is within the design range. In research [17], the aluminum plate has been repaired with a single-sided composite patch, and the results have shown that fatigue life is increased by at least four times, and in some cases, it can increase fatigue life by more than four times. Also, the research results [17] have shown that the difference in crack length on the patched side and the non-repaired side for high-

thickness plates is greater than with thinner plates. The use of double-sided patches in repair with composite patches further reduces the stress intensity factor compared to single-sided patches [18]. In the one-sided repair of plates with composite patches, bending occurs out of the plate and affects the efficiency of the patch in repair [3]. The off-plane bending created in one-sided repair causes the plate to also experience bending stresses [13]. One-sided repair causes the stress intensity factor to increase on the unrepaired side, and with increasing patch thickness, the difference in stress intensity factor on both sides increases [19]. In the study [16], which examined four patches of boron, graphite, glass, and carbon, the results showed that the boron patch has the highest efficiency and glass has the lowest efficiency, and the two patches of carbon and graphite have almost the same behavior and efficiency.

In this research, the effect of composite patch dimensions and adhesive thickness on the efficiency of one-sided and two-sided repair of the cracked plate is investigated. To investigate the effect of the dimensions of the composite patch as well as the thickness of the adhesive on the stress intensity factor in the repair of cracked aluminum plate, boron, graphite, carbon, and glass epoxy patches have been used, and by considering different thicknesses, lengths, and widths for each of the composite patches and different thicknesses for the adhesive, the effect of increasing each of the parameters on the stress intensity factor in one-sided and two-sided repair has been investigated.

2- Problem definition and finite element modeling

To investigate the effect of composite patch dimensions and adhesive thickness on repair efficiency, first, a 105 x 65 mm rectangular cracked plate with a thickness of 3 mm made of 2024 aluminum alloy with a modulus of elasticity 74 Gpa and Poisson's ratio of 0.33 in Abaqus software is modeled in 3D. The crack length is 9 mm. A tensile load of 15 MPa is applied to the top and bottom edges of the plate. Then, to repair the plate with composite patches, 30 × 30 mm square composite patches with a thickness of 1 mm made of boron, graphite, carbon, and glass epoxy have been used. The adhesive used to repair the plate is made of FM 73, which is a thin film with a thickness of 0.1 mm. The properties of the composite patches and adhesives used in this study are presented in Table 1. The properties of patches are taken from references [20-22]. Figure 1 shows the geometry studied in this research. The contour integral method was used to calculate the stress intensity factor at the crack tip. 8-node linear three-dimensional elements (C3D8R) are used in the modeling. The model of the cracked aluminum plate repaired with a composite patch in Abaqus software as well as the mesh around the crack tip is shown in Figure 2.

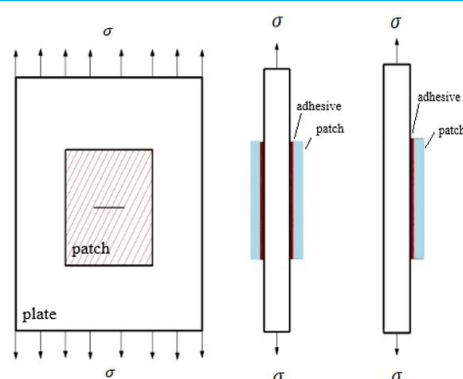
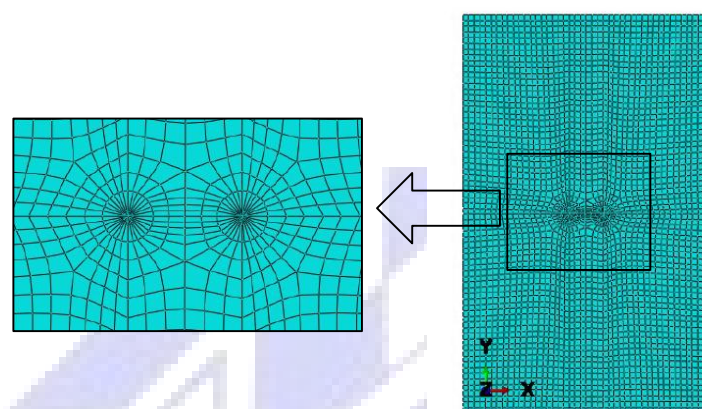
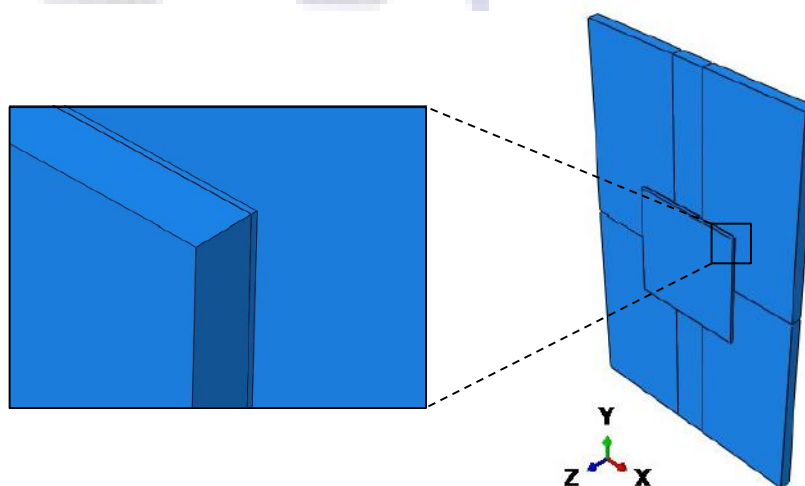


Figure 1- Geometry of repairing cracked plate with one-sided and two-sided composite patch



a) Cracked plate meshing and mesh around the crack tip



b) Finite element modeling of cracked plate repaired with composite patch in Abaqus software

Figure 2 - Finite element modeling of cracked plate repaired with composite patch in Abaqus software and mesh around the crack tip

Table 1- Properties of composite patches and adhesives (E and G in Gpa)

Material	E_{11}	E_{22}	E_{33}	G_{12}	G_{13}	G_{23}	ν_{12}	ν_{13}	ν_{23}
Graphite-epoxy	172.4	10.34	10.34	4.82	4.82	3.10	0.3	0.3	0.18
Glass-epoxy	27.82	5.83	5.83	2.56	2.56	2.24	0.31	0.31	0.41
Boron-epoxy	208.1	25.44	25.44	7.24	7.24	4.94	0.1677	0.1677	0.035
Carbon-epoxy	134	10.3	10.3	5.5	5.5	2.3	0.33	0.33	0.53
Adhesive-FM 73	2.21	-	-	-	-	-	0.43	-	-

3- Finite element analysis validation

To validate the results of finite element analysis, we compare the results obtained for the stress intensity factor of finite element analysis with the results obtained from theoretical relation (1).

$$K_I = Y\sigma\sqrt{\pi a} \quad (1)$$

In relation (1), $\sigma = 15$ Mpa is the stress applied to the plate, $a = 4.5$ mm is half the crack length and Y is the geometric coefficient obtained from the following equation (2).

$$Y = \sqrt{\sec(\pi a/w)} \quad (2)$$

In Equation (2), $a = 4.5$ mm is half the crack length, and $w = 65$ mm is the plate width. The stress intensity factor obtained from the theoretical relation $K_I = 57.07$ Mpa. $\sqrt{\text{mm}}$ and also the stress intensity factor obtained from finite element analysis is $K_I = 59.17$ Mpa. $\sqrt{\text{mm}}$, and the difference in stress intensity factor of both methods is 3.68% and this shows that the modeling performed is of acceptable accuracy.

4- Analysis of the obtained results

In this section, we review and analyze the results obtained from the analysis of finite elements of cracked aluminum plate repaired with one-sided and two-sided composite patches of boron, graphite, carbon, and glass epoxy in Abaqus software. Figure 4 shows the changes in stress intensity factor along the thickness of cracked aluminum plate repaired with composite patches. As we can see in Figure 4, in the one-sided repair mode for all composite patches, the stress intensity factor decreases on the repaired side and increases on the unrepaired side, due to off-plate bending. Therefore, to compare the stress intensity factor in one-sided repair mode with the unrepaired state and calculate the percentage of stress intensity factor reduction, we use the average stress intensity factor in terms of thickness. Off-plane bending for the repaired plate with boron and glass epoxy is shown in Figure 3. However, in double-sided repair, due to the symmetry of the composite patches on both sides of the plate, unlike the repair mode with one-sided patches, the stress intensity factor is reduced by almost one ratio and the stress intensity factor does not change much in the plate thickness.

According to Figure 4, in one-sided and two-sided repair mode, the highest reduction in stress intensity factor is related to boron epoxy patch, which reduces the stress intensity factor in one-sided and two-sided states by 23.33% and 73.7%, respectively, also, the least effect on

the stress intensity factor in the two repair modes is related to the glass epoxy patch, which reduces the stress intensity factor to a lesser extent than other patches and has the lowest efficiency among the studied patches. Glass patch in one-sided and two-sided mode reduced the stress intensity factor by 13.5% and 32.8%, respectively. In the case of one-sided repair, the graphite and carbon patches reduce the stress intensity factor almost as much as the boron patch and have a difference of about 1 to 2% with the boron epoxy patch. In the case of one-sided repair, the carbon and graphite patch reduces the stress intensity factor by 21.33% and 22.3%, respectively, and also in the two-sided repair, the carbon and graphite patch reduces the stress intensity factor by 65.32% and 69.78%, respectively.

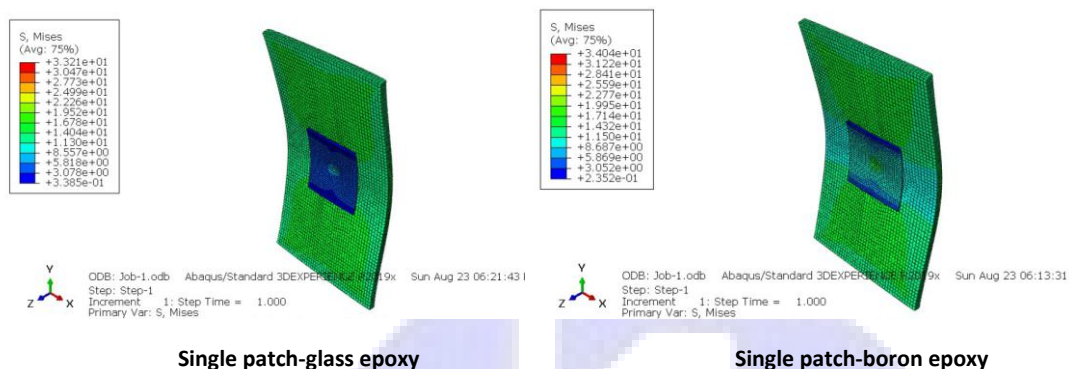


Figure 3- Off-plate bending in one-sided repair with boron and glass epoxy composite patch

According to the results obtained from the analysis of patches in one-sided and two-sided repair for all four composite patches studied in this study, the use of the two-sided patches in the repair of the cracked plate has reduced the stress intensity factor more than one-sided repair, and for example, for the boron epoxy patch, which has the highest efficiency among the patches under study, in two-sided repair, the stress intensity factor is reduced by 50% more than in one-sided repair mode. Therefore, according to the results, for repairing cracks with composite patches, it is better to use two-sided repair, because as it was shown, it reduces the stress intensity factor more than one-sided repair. However, in cases where there is no possibility of two-sided repair for cracks, such as repairing cracks in the hull of a ship or aircraft, in such cases where it is not possible to repair both sides, we repair the crack with one-sided patches. Therefore, whether it is possible to repair on both sides or not, according to the studies performed for all four composite patches, it is better to use boron epoxy patch, because it has the highest efficiency among patches in two modes of one-sided and two-sided repair.

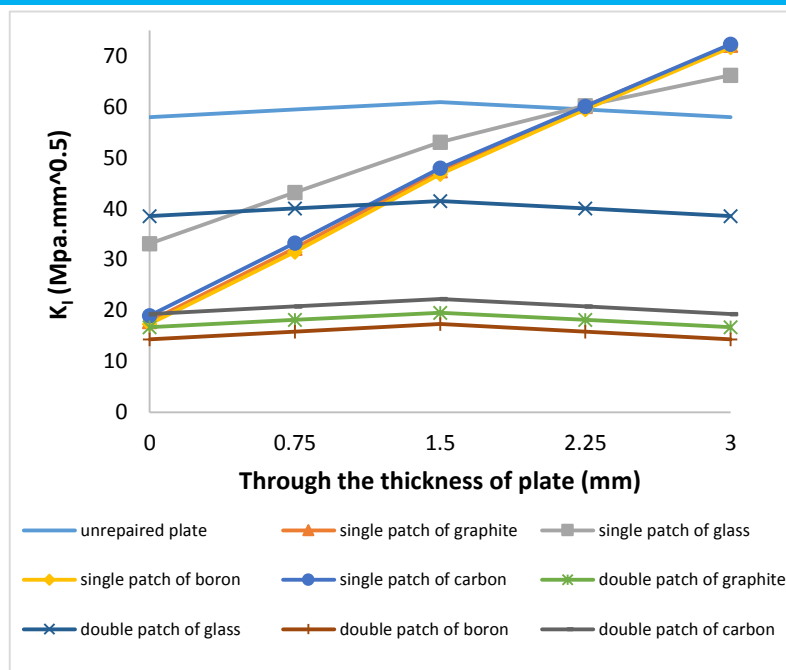


Figure 4 - Variation of stress intensity factor along the thickness of the repaired plate with composite patches

4-1- The effect of patch thickness on stress intensity factor

To investigate the effect of patch thickness on patch efficiency, we use patches with thickness of 1, 1.5, 2, 2.5 and 3 mm. In this study, by keeping the length and width of the patches constant as well as the thickness of the adhesive and only by changing the thickness of the composite patches, the results of the effect of increasing the patch thickness on the efficiency of each of the composite patches of boron, graphite, carbon and glass epoxy in one-sided and two-sided repair mode are shown in Figure 5. The length and width of the patch in this study are the same and equal to 30 mm. According to Figure 5, which shows the changes in the average stress intensity factor in terms of patch thickness, in one-sided repair mode with increasing the thickness of glass epoxy patch from 1 to 1.5 mm, we see a slight change in reducing the stress intensity factor and from 1.5 mm Then, by increasing the thickness of the glass epoxy patch, it does not have much effect on reducing the stress intensity factor, but for boron, carbon and graphite patches, the stress intensity factor decreases with increasing the thickness of the patches. However, in the case of two-sided repair for all four patches, with increasing the thickness of the patches, the stress intensity factor decreases and the ratio of decreasing the stress intensity factor with increasing the thickness of the patches in two-sided repair is higher than one-sided repair. That is, the effect of increasing the thickness of the patch by 0.5 mm in two-sided repair is more than one-sided repair for all patches, and further reduces the stress intensity factor in two-sided repair.

Also, according to Figure 5, we see that the effect of increasing the thickness of the patches by 0.5 mm in two modes of one-sided and two-sided repair and especially in two-sided repair on the stress intensity factor reduction is gradually reduced, thus, increasing the thickness of patches by 0.5 mm from 1 to 1.5 has a greater effect on reducing the stress intensity factor

than increasing the thickness of patches from 1.5 to 2 mm, and this continues with increasing the thickness of the patches, and the more we increase the thickness of the patch, the lower the percentage of stress intensity factor decreases from the previous stage, and it seems that from somewhere onwards, increasing the thickness does not have much effect on the stress intensity factor. This means that increasing the patch thickness to a certain value has a good effect on the stress intensity factor and increasing the thickness of the patches can be used as a parameter with which the stress intensity factor can be greatly reduced, but after that, it does not have a significant effect and the percentage reduction of stress intensity factor does not change much.

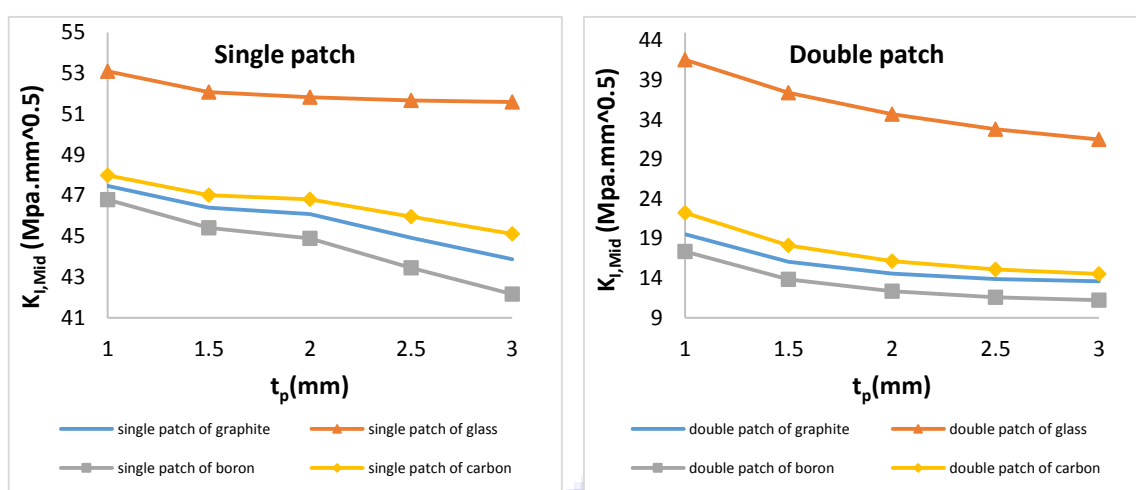


Figure 5 - Variation of average stress intensity factor in terms of patch thickness for plate repaired with composite patches

As mentioned, after a few steps of increasing the thickness of the patches, the ratio of stress intensity factor reduction, decreases compared to the initial stages and increasing the patch too much does not significantly change the stress intensity factor and only the patch thickness increases and in some industries, this thickness may not be suitable, and especially in industries and aerospace applications, thick patches may interfere with the aerodynamics of structures, therefore, in such cases where there is a limit to the thickness of the patch, the thickness of the patch can not be considered too much, and there must be an intermediate between how much we increase the thickness of the patch and what the desired reduction is, but when there is no limit to the thickness of the patch, it is better to use a thick patch, because in general, increasing the thickness of the patch reduces the stress intensity factor.

4-2- The effect of patch length on stress intensity factor

The effect of patch length on patch efficiency in cracked plate repair is investigated by considering different lengths of 30, 40, 50, and 60 mm for composite patches in one-sided and two-sided repair. The different lengths intended for patches are shown in Figure 6. In this study, the thickness and width of the patch as well as the thickness of the adhesive are constant and the only variable is the length of the patch. In this study, the thickness and width of the patch are equal to 1 and 30 mm, respectively. The changes in the average stress intensity factor in terms of patch length for cracked plate repaired with composite patches are shown in Figure 7. According to Figure 7, increasing the length of patches in one-sided

and two-sided repair increases the stress intensity factor and as we can see in Figure 7, the amount of increase in stress intensity factor with increasing the length of patches in two-sided repair is more than one-sided repair, that is, increasing the length of patches in one-sided repair has less effect on increasing the stress intensity factor than two-sided repair. Therefore, according to the explanations provided, increasing the length of patches not only does not reduce the stress intensity factor, but also increases the stress intensity factor, and especially in two-sided repair, it increases the stress intensity factor more than one-sided repair, so increasing patch length is not a good way to reduce the stress intensity factor.

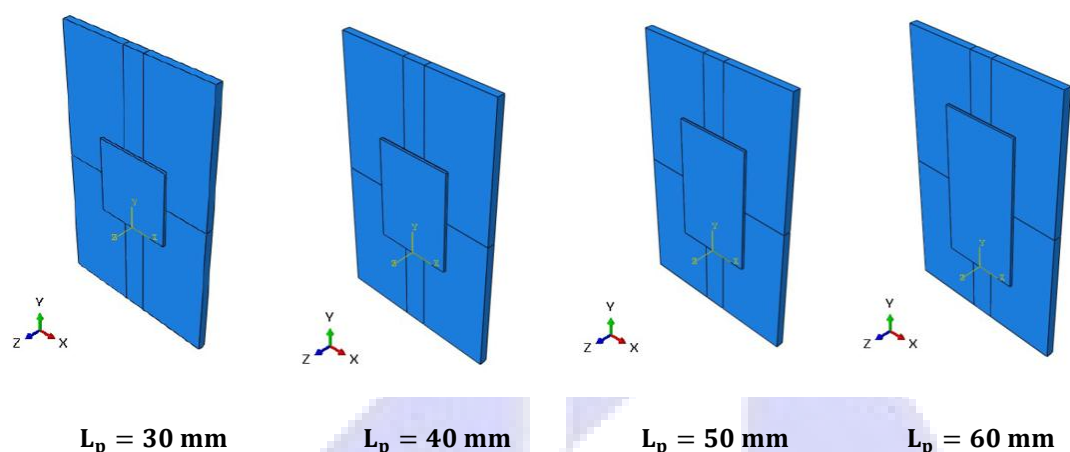


Figure 6- Different lengths intended for composite patches in one-sided and two-sided repair

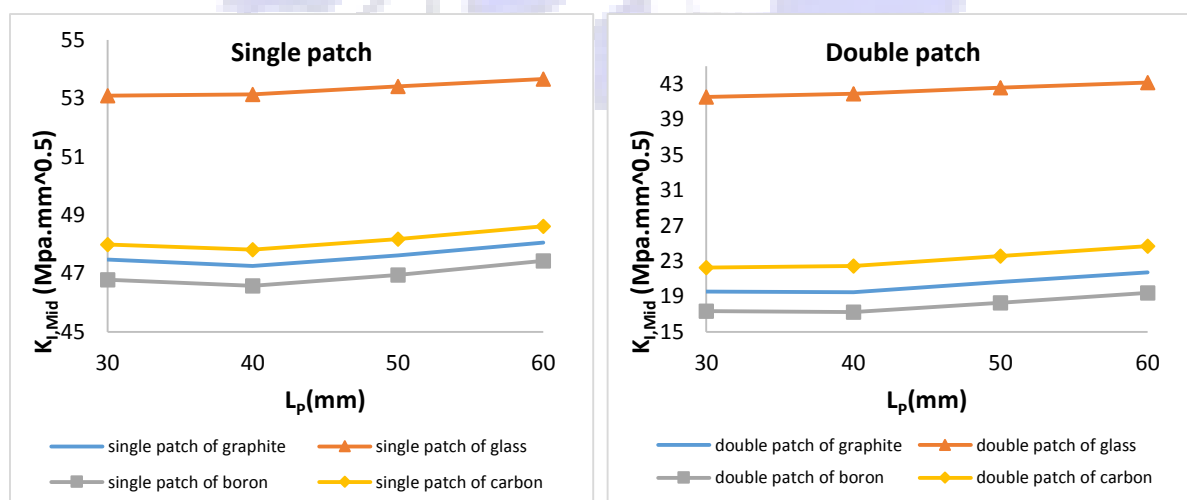


Figure 7 - Variation of average stress intensity factor in terms of patch lengths for plate repaired with composite patches

4-3- The effect of patch width on stress intensity factor

To investigate the effect of patch width on patch efficiency, we use composite patches with different widths of 30, 40, 50, and 60 mm and by keeping the length and thickness of the

patch and the thickness of the adhesive constant, we repair cracked plate with composite patches in both one-sided and two-sided modes, and the results of the analysis are shown in Figure 9. In this study, the thickness and length of the patch are equal to 1 and 30 mm, respectively. The different widths intended for patches are shown in Figure 8. According to Figure 9, in one-sided repair mode, increasing the patch width has no effect on the efficiency of the glass epoxy patch. but for boron, carbon, and graphite patches, it increases the stress intensity factor, therefore, in one-sided repair, increasing the patch width is not suitable to reduce the stress intensity factor.

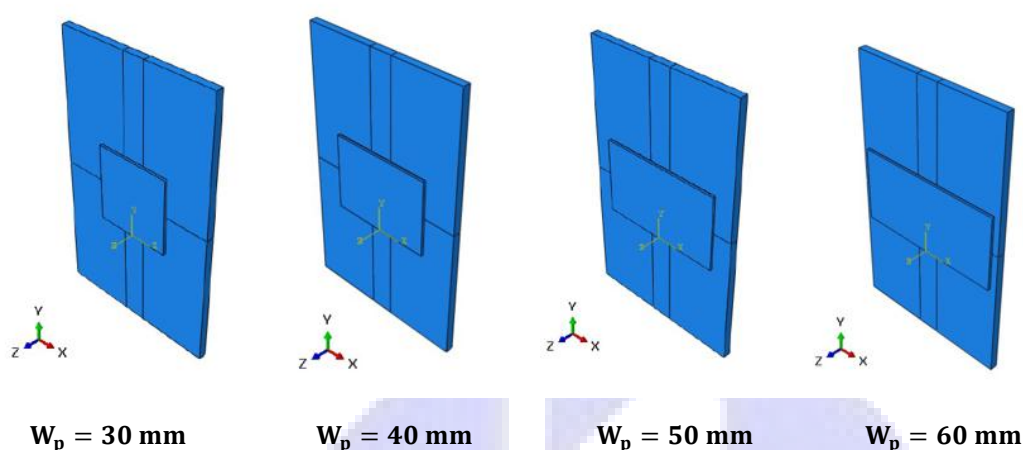


Figure 8- Different widths intended for composite patches in one-sided and two-sided repair

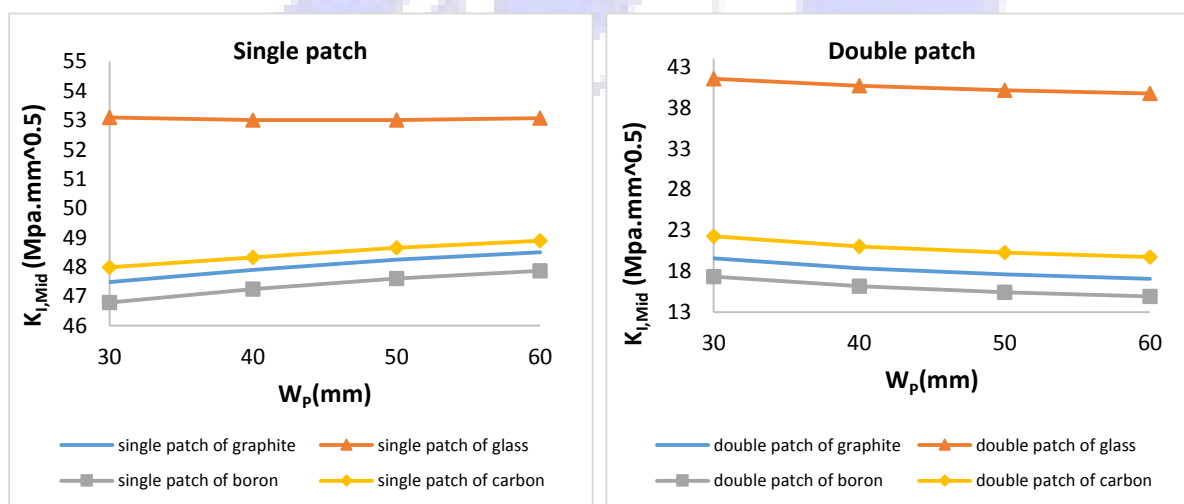


Figure 9 - Variation of average stress intensity factor in terms of patch widths for plate repaired with composite patches

But keep in mind that when the width of the composite patch is small, when the crack grows, the crack will soon reach the edges of the composite patch and, therefore, the patch with a wide width prevents the crack from reaching the edges of the patch soon and causes the crack to be under the patch for a long time. Therefore, the advantage of considering a wide patch in one-sided repair is that it leaves the crack under the patch for a long time, otherwise, except for the glass epoxy patch, it increases the stress intensity factor. However, in the case of two-sided repair for all four composite patches of boron, graphite, carbon, and glass epoxy,

increasing the width of the patch reduces the stress intensity factor, and in this case, increasing the patch width is a good way to reduce the stress intensity factor.

4-4- Effect of adhesive thickness on stress intensity factor

In this section, the effect of increasing the adhesive thickness on the repair efficiency of the cracked plate with composite patches is investigated. In this study, the thickness of the adhesive layer is 0.1, 0.2, 0.3, 0.4, and 0.5 mm, respectively. The results of the effect of increasing the adhesive thickness on the repair efficiency of the cracked plate in one-sided and two-sided repair mode with boron, carbon, graphite, and glass epoxy composite patches are shown in Figure 10. According to Figure 10 in both one-sided and two-sided repair modes, increasing the adhesive thickness increases the stress intensity factor and thus reduces the repair efficiency. However, in both one-sided and two-sided repairs, the amount of increase in stress intensity factor with increasing adhesive thickness in repair with glass-epoxy patch is less than the other three patches, for example, in one-sided repair with boron epoxy patch, increasing the adhesive thickness from 0.1 to 0.3 mm has increased the stress intensity factor by 4.96%, but in glass repair, increasing the same amount of adhesive thickness has only increased the stress intensity factor by 1.75%. Also according to Figure 10, the amount of increase in stress intensity factor with increasing adhesive thickness in two-sided repair is much more than one-sided repair, so that in two-sided repair with carbon epoxy patch by tripling the thickness of the adhesive (0.1 to 0.3), the stress intensity factor has increased by 18.63%, but in one-sided repair, increasing the same thickness has only increased the stress intensity factor by 4.23% and as we can see, in two-sided repair, the stress intensity factor is increased by approximately 4.4 times the one-sided repair mode. Increasing the adhesive thickness from 0.1 to 0.4 in one-sided and two-sided repair with graphite-epoxy patch increased the stress intensity factor by 6.28% and 34.20%, respectively, which increased the stress intensity factor much more in two-sided repair (5.4 times).

The results also show that by increasing the thickness of the adhesive in the one-sided and two-sided repair of the plate with all four composite patches, the maximum Von Mises stress in the adhesive layer decreases and thus increases the durability of the patch. so, although increasing the thickness of the adhesive has increased the stress intensity factor, but on the other hand, has increased the patch durability. The maximum Von Mises stress in adhesive layers with a thickness of 0.1, 0.3, and 0.5 mm in the one-sided repair of the cracked plate with boron epoxy patch is shown in Figure 11. According to Figure 11, by tripling the thickness of the adhesive (0.1 to 0.3), the maximum von Mises stress in the adhesive layer is reduced by 38.53%.

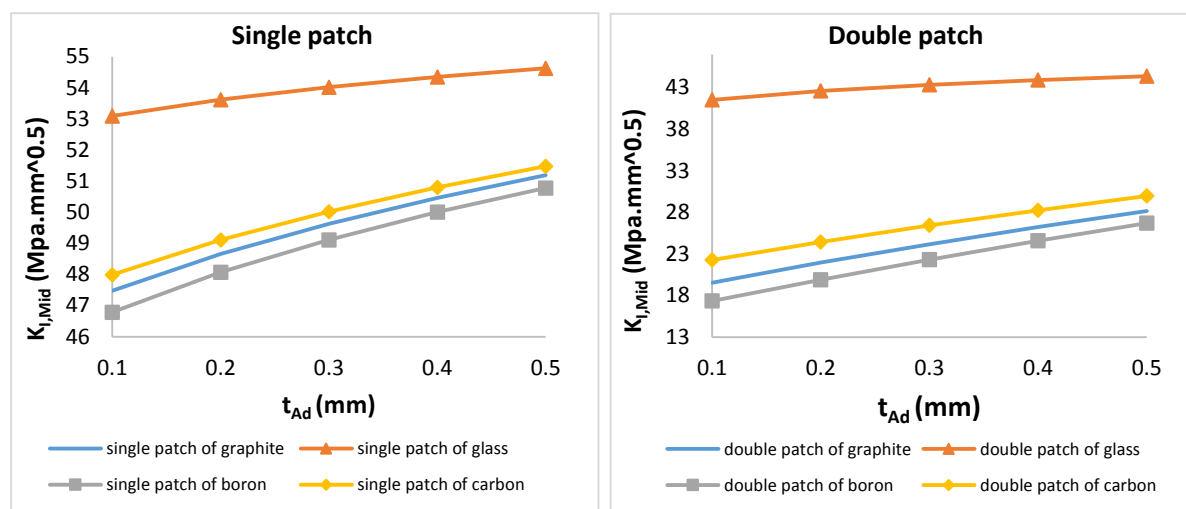


Figure 10 - Variation of average stress intensity factor in terms of adhesive thickness for plate repaired with composite patches

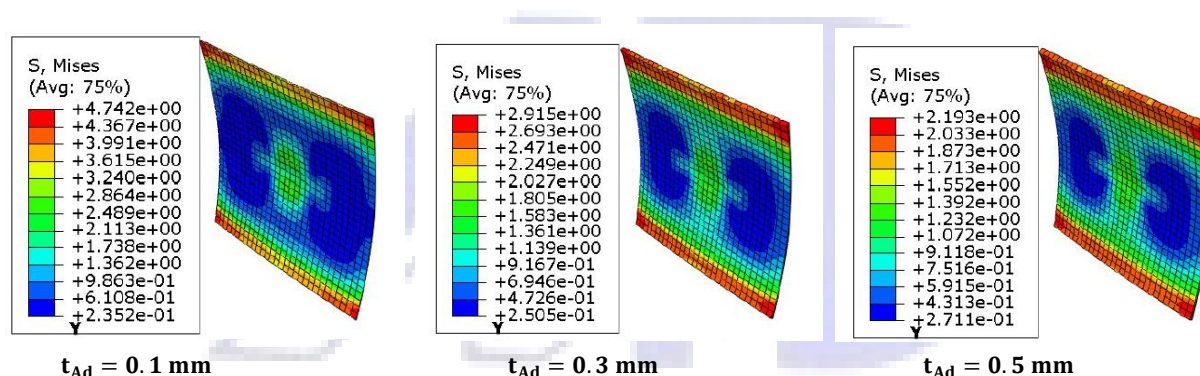


Figure 11- Maximum Von Mises stress in the adhesive layer with different thicknesses in the one-sided repair of the plate with boron-epoxy patch

5- Summary and conclusion

In this study, in order to investigate the effect of composite patch dimensions and adhesive thickness on repair efficiency, cracked aluminum plate with boron, graphite, carbon, and glass epoxy composite patches has been repaired in both one-sided and two-sided modes and by considering different thicknesses, lengths, and widths for each of the patches and different thicknesses for the adhesive, the effect of each parameter on the repair efficiency of cracked plate has been investigated and the most important research results are as follows:

- 1- In two modes of one-sided and two-sided repair, the highest reduction of stress intensity factor is related to the boron-epoxy patch and the lowest is related to the glass-epoxy patch.
- 2- boron-epoxy patch in one-sided and two-sided repair modes reduces the stress intensity factor by 23.33% and 73.7%, respectively, and compared to glass-epoxy patch in one-sided

and two-sided repair modes, it has caused a further reduction of 9.83% and 40.9% in the stress intensity factor, respectively.

3- In one-sided repair, carbon, and graphite-epoxy patches reduce the stress intensity factor almost to the extent of boron patches and have a very small difference of about 1 and 2%.

4- Double-sided repair with carbon and graphite-epoxy patches reduces 65.32% and 69.32% of stress intensity factor, respectively.

5- Increasing the thickness of patches in one-sided and two-sided repair reduces the stress intensity factor and its effect is more in two-sided repair.

6- In one-sided repair mode, increasing the thickness of the glass-epoxy patch has a small effect on reducing the stress intensity factor compared to boron, graphite, and carbon patches.

7- Increasing the length of patches in one-sided and two-sided repair not only does not reduce the stress intensity factor, but also increases it, so it is better not to consider the length of patches too much.

8- Increasing the width of the patch in one-sided repair with glass-epoxy patch has no effect on the stress intensity factor, but for boron, carbon, and graphite patches, it increases the stress intensity factor.

9- Increasing the width of patches in two-sided repair reduces the stress intensity factor, so in two-sided repair to further reduce the stress intensity factor, it is better to use a wide width patch.

10- In both one-sided and two-sided repair modes, increasing the thickness of the adhesive has increased the stress intensity factor, but on the other hand, with increasing the thickness of the adhesive, the maximum Von Mises stress in the adhesive layer has decreased, therefore, the durability of the patch increases with increasing the thickness of the adhesive.

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7- Appendix

Notation

E	Young's modulus
G	Shear modulus
K_I	Mode I Stress intensity factor
$K_{I, Mid}$	Mode I average stress intensity factor
L_p	Patch length
W_p	Patch width
Y	Geometric factor
$2a$	Crack length
t_p	Patch thickness
t_{Ad}	Adhesive thickness
ν	Poisson's ratio

